21 April 2020 2A&2B

MATH 306 Workshop

Important Theorems: (you should also review all the definitions)
Span: 2.21 (Linear Dependence Lemma)
Bases: 2.29, 2.31, 2.32, 2.33, 2.34

1. What is the definition of a **basis**?

2. Recall that $\mathcal{P}(\mathbf{F})$ is the set of all polynomials with coefficients in **F**.

   a. What does $\mathcal{P}_3(\mathbf{F})$ stands for?

   b. Give an example of a basis of $\mathcal{P}_3(\mathbf{F})$.

3. Explain why there does not exist a list of six polynomials that is linearly independent in $\mathcal{P}_4(\mathbf{F})$.

4. Prove that the list (1, 0, 0), (1, 1, 0), (1, 1, 1) spans $\mathbf{F}^3$.

5. Let U be the subspace of $\mathbf{F}^5$ defined by:

   $U = \{ (x_1, x_2, x_3, x_4, x_5) \in \mathbb{F}^5 \mid x_2 = x_1, x_3 = 2x_4 - 3x_5 \}$

   (a) Find a basis for U and prove your answer is a basis.

   (b) Let W be the subspace of $\mathbf{F}^5$ defined by:

   W = span{(1, 0, 0, 0, 0), (0, 0, 1, 0, 0)}

   Show that $\mathbf{F}^5 = U \oplus W$.

6. Prove or disprove: there exists a basis p_0, p_1, p_2, p_3 of $\mathcal{P}_3(\mathbf{F})$ such that none of the polynomials p_0, p_1, p_2, p_3 has degree 2.

7. Let U be the subspace of $\mathbf{C}^5$ defined by:

   U = $\{(z_1, z_2, z_3, z_4, z_5) \in \mathbf{C}^5 : 6z_1 = z_2 \text{ and } z_3 + 2z_4 + 3z_5 = 0\}$

   (a) Find a basis for U.

   (b) Extend the basis in part (a) to a basis of $\mathbf{C}^5$.

   (c) Find a subspace W of $\mathbf{C}^5$ such that $\mathbf{C}^5 = U \oplus W$.

Proving techniques

1. Proving uniqueness
2. Proving "for all" statements
3. Proving biconditional statements